5 ASSESS AND RETEACH

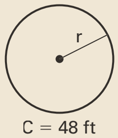
Daily Homework Quiz

Transparency Available

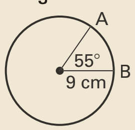
Find the indicated measure.

1. Circumference **about 81.68 in.**

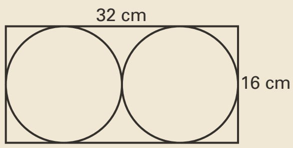
2. Radius **about 7.64 ft**



3. length of $\overset{\frown}{AB}$ **8.64 cm**

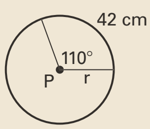


4. Find the total circumference of the circles. **100.53 cm**

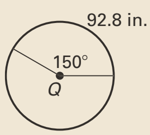


Find the indicated measure.

5. Radius **about 21.88 cm**



6. Circumference **about 222.72 in.**



Online Quiz

Available at classzone.com

Diagnosis/Remediation

- Practice A, B, C in Chapter 11 Resource Book, pp. 49–54
- Study Guide in Chapter 11 Resource Book, pp. 55–56
- Practice Workbook, pp. 214–216
- @HomeTutor

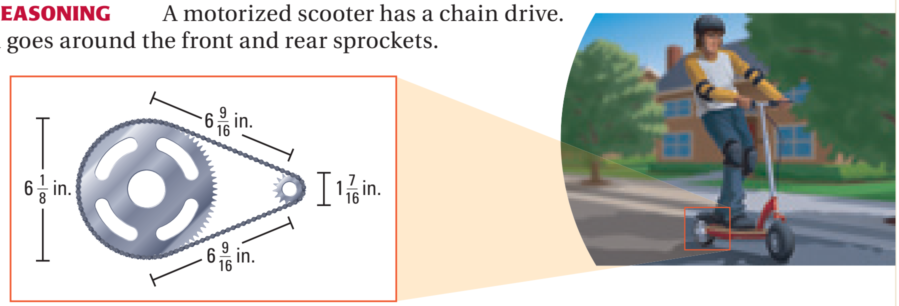
Challenge

Additional challenge is available in the Chapter 11 Resource Book, p. 59.

38a. About 25 in.; since the chain only touches about half of each sprocket, determine half the circumference of each sprocket and add twice the sum of the long segments between the sprockets.

38b. About 46 teeth; since the chain only touches about half of each sprocket, determine half of the teeth of each sprocket.

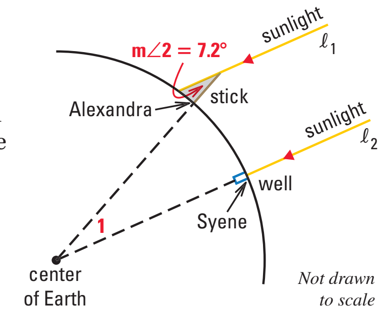
B **38.** **TAKS REASONING** A motorized scooter has a chain drive. The chain goes around the front and rear sprockets.



a. About how long is the chain? *Explain.*

b. Each sprocket has teeth that grip the chain. There are 76 teeth on the larger sprocket, and 15 teeth on the smaller sprocket. About how many teeth are gripping the chain at any given time? *Explain.*

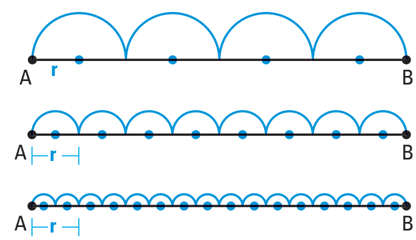
39. **SCIENCE** Over 2000 years ago, the Greek scholar Eratosthenes estimated Earth's circumference by assuming that the Sun's rays are parallel. He chose a day when the Sun shone straight down into a well in the city of Syene. At noon, he measured the angle the Sun's rays made with a vertical stick in the city of Alexandria. Eratosthenes assumed that the distance from Syene to Alexandria was equal to about 575 miles.



Find $m\angle 1$. Then estimate Earth's circumference. **7.2°; 28,750 mi**

C **CHALLENGE** **Suppose $\overline{AB}$ is divided into four congruent segments, and semicircles with radius r are drawn.**

40. What is the sum of the four arc lengths if the radius of each arc is r? **$4\pi r$**



41. Suppose that $\overline{AB}$ is divided into n congruent segments and that semicircles are drawn, as shown. What will the sum of the arc lengths be for 8 segments? for 16 segments? for n segments? *Explain* your thinking. **$4\pi r$; $4\pi r$; $4\pi r$; the length is the same, just allocated differently.**

MIXED REVIEW FOR TAKS

TAKS PRACTICE at classzone.com

REVIEW
TAKS Preparation p. 288;
TAKS Workbook

42. **TAKS PRACTICE** Which equation best describes the functional relationship between x and y in the table below? *TAKS Obj. 2* **C**

x	−2	−1	1	3
y	17	3	−1	−53

Ⓐ $y = -14x - 11$ Ⓑ $y = 4x^2 - 1$

Ⓒ $y = -2x^3 + 1$ Ⓓ $y = -2x + 1$

EXTRA PRACTICE for Lesson 11.4, p. 916 **ONLINE QUIZ** at classzone.com

EXTRA MIXED REVIEW FOR TAKS

Which equation best describes the functional relationship between x and y in the table? *TAKS Obj. 2* **C**

x	−3	−1	2	4
y	15	−1	5	29

Ⓐ $y = -5x$ Ⓑ $y = 3x^2 - 12$

Ⓒ $y = 2x^2 - 3$ Ⓓ $y = 2x + 2$

Extension

Use after Lesson 11.4

Geometry on a Sphere

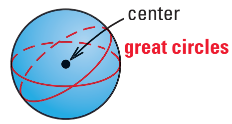
TEKS *G.1.A, G.1.B, G.1.C, G.9.D*

GOAL Compare Euclidean and spherical geometries.

Key Vocabulary
- **great circle**

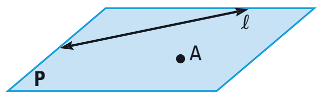
In Euclidean geometry, a plane is a flat surface that extends without end in all directions, and a line in the plane is a set of points that extends without end in two directions. Geometry on a sphere is different.

In *spherical geometry*, a plane is the surface of a sphere. A line is defined as a **great circle**, which is a circle on the sphere whose center is the center of the sphere.



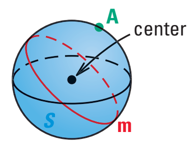
KEY CONCEPT — *For Your Notebook*

Euclidean Geometry



Plane P contains line ℓ and point A not on the line ℓ.

Spherical Geometry



Sphere S contains great circle m and point A not on m. Great circle m is a line.

HISTORY NOTE

Spherical geometry is sometimes called *Riemann geometry* after Bernhard Riemann, who wrote the first description of it in 1854.

Some properties and postulates in Euclidean geometry are true in spherical geometry. Others are not, or are true only under certain circumstances. For example, in Euclidean geometry, Postulate 5 states that through any two points there exists exactly one line. On a sphere, this postulate is true only for points that are not the endpoints of a diameter of the sphere.

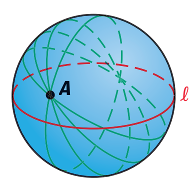
EXAMPLE 1 Compare Euclidean and spherical geometry

Tell whether the following postulate in Euclidean geometry is also true in spherical geometry. Draw a diagram to support your answer.

Parallel Postulate: If there is a line ℓ and a point A not on the line, then there is exactly one line through the point A parallel to the given line ℓ.

Solution

Parallel lines do not intersect. The sphere shows a line ℓ (a great circle) and a point A not on ℓ. Several lines are drawn through A. Each great circle containing A intersects ℓ. So, there can be no line parallel to ℓ. The parallel postulate is not true in spherical geometry.



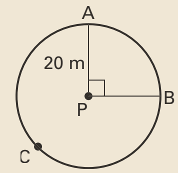
1 PLAN AND PREPARE

Warm-Up Exercises

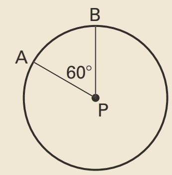
1. Find the length of a 60° arc in a circle with radius 8 meters. **about 8.38 m**
2. Find the exact value of x if $\frac{12}{2\pi \cdot 8} = \frac{x}{360}$. $\frac{270}{\pi}$

Find the exact length of the arc.

3. $\overset{\frown}{ACB}$ **300π m**



4. $\overset{\frown}{AB}$ **$13\frac{1}{3}$ cm**



Circumference = 80 cm

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 3, p. 719

On the surface of a sphere, how can you identify the shortest distance between two points? **Tell students they will learn how to answer this question by identifying a great circle on the sphere.**

Resource Guide

See p. 718B for a complete list of resources for:
- differentiating instruction
- integrating technology

TEKS **G.1.A Geometric Structure:** Develops an awareness of the structure of a mathematical system, connecting definitions, postulates, and theorems.
G.1.B Geometric Structure: Recognizes the historical development of geometric systems and knows mathematics is developed for a variety of purposes.
G.1.C Geometric Structure: Compares and contrasts the structures and implications of Euclidean and non-Euclidean geometries.
G.9.D Congruence and Geometry of Size: Analyzes the characteristics of polyhedra and other 3-D figures and their component parts based on explorations and models.

TAKS **Objective 7:** Demonstrates an understanding of 3-D geometric representations.

3 TEACH

Extra Example 1

Tell whether the following postulate in Euclidean geometry is also true in spherical geometry. Draw a diagram to support your answer. Postulate 5: Through any two points there exists exactly one line.

False; if the two points are the endpoints of a diameter, many great circles contain the two points.

Extra Example 2

The diameter of a sphere is 24 units, and for two points A and B on the sphere, $m\widehat{AB} = 135°$. Find the distance between A and B on the minor arc and on the major arc of a great circle. **9π units; 15π units**

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: On the surface of a sphere, how can you identify the shortest distance between two points?

- **A line on a sphere is a great circle.**
- **Some postulates from Euclidean geometry are not true for spherical geometry.**

Find the minor arc length on the great circle that contains the two points.

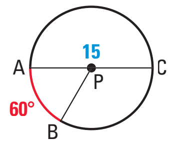
4 PRACTICE AND APPLY

Teaching Strategy

Exercise 3 Suggest that students draw sketches to support their conjecture for this exercise.

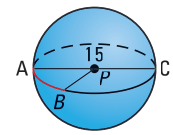
DISTANCES In Euclidean geometry, there is exactly one distance that can be measured between any two points. On a sphere, there are two distances that can be measured between two points. These distances are the lengths of the major and minor arcs of the great circle drawn through the points.

READ DIAGRAMS
The diagram below is a cross section of the sphere in Example 2. It shows $\widehat{AB}$ and $\widehat{ACB}$ on a great circle.



EXAMPLE 2 Find distances on a sphere

The diameter of the sphere shown is 15, and $m\widehat{AB} = 60°$. Find the distances between A and B.



Solution

Find the lengths of the minor arc $\widehat{AB}$ and the major arc $\widehat{ACB}$ of the great circle shown. In each case, let x be the arc length.

$$\frac{\text{Arc length of } \widehat{AB}}{2\pi r} = \frac{m\widehat{AB}}{360°} \qquad \frac{\text{Arc length of } \widehat{ACB}}{2\pi r} = \frac{m\widehat{ACB}}{360°}$$

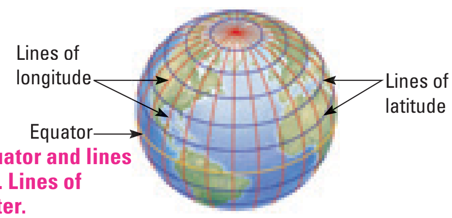
$$\frac{x}{15\pi} = \frac{60°}{360°} \qquad \frac{x}{15\pi} = \frac{360° - 60°}{360°}$$

$$x = 2.5\pi \qquad x = 12.5\pi$$

▶ The distances are 2.5π and 12.5π.

PRACTICE

1. **WRITING** Lines of latitude and longitude are used to identify positions on Earth. Which of the lines shown in the figure are great circles. Which are not? *Explain* your reasoning.
Equator and longitude lines; latitude lines; the equator and lines of longitude have the center of Earth as the center. Lines of latitude do not have the center of Earth as the center.



EXAMPLE 1 on p. 753 for Exs. 2–3

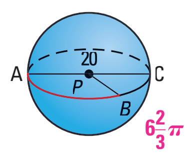
2. **COMPARING GEOMETRIES** Draw sketches to show that there is more than one line through the endpoints of a diameter of a sphere, but only one line through two points that are *not* endpoints of a diameter. **See margin.**

3. **COMPARING GEOMETRIES** The following statement is true in Euclidean geometry: If two lines intersect, then their intersection is exactly one point. Rewrite this statement to be true for lines on a sphere. *Explain.*
If two lines intersect then their intersection is exactly 2 points.

EXAMPLE 2 on p. 754 for Exs. 4–6

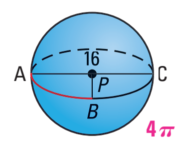
FINDING DISTANCES **Use the diagram and the given arc measure to find the distances between points A and B. Leave your answers in terms of π.**

4. $m\widehat{AB} = 120°$



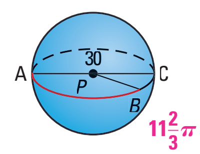
$6\frac{2}{3}\pi$

5. $m\widehat{AB} = 90°$



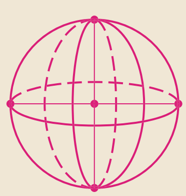
4π

6. $m\widehat{AB} = 140°$

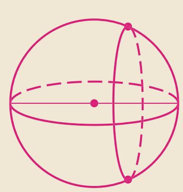


$11\frac{2}{3}\pi$

2.



More than 1 line through endpoints of diameter on a sphere



Only 1 line through 2 points not endpoints of diameter